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BLACK HOLE'S SHADOW AND ITS MEASUREMENT

In this study, we looked into ideas like measuring black hole shadows, a generic formula for calculating the shadow radius of symmetric spherical black holes, and the effect of symmetric gravity on black hole shadow. We first acquire the zero geodesic equations in order to compute the black hole's shadow under symmetric gravity. With these formulas in hand, we can now determine the shadow radius for the black hole metric. We will also briefly discuss the derivation of the shadow formula for any given spherically symmetric black hole. Theoretically, it is evident that other compact objects, such as empty singularities, in addition to black holes, can cast shadows. We demonstrate how a new technique for detecting the shadows thrown by a twin black hole can allow astronomers to learn more about these gigantic systems. We give an analytical computation of shadows for singularities in the nothingness of space-time.

Key words: *Black hole, naked singularity, shadow.*

Introduction

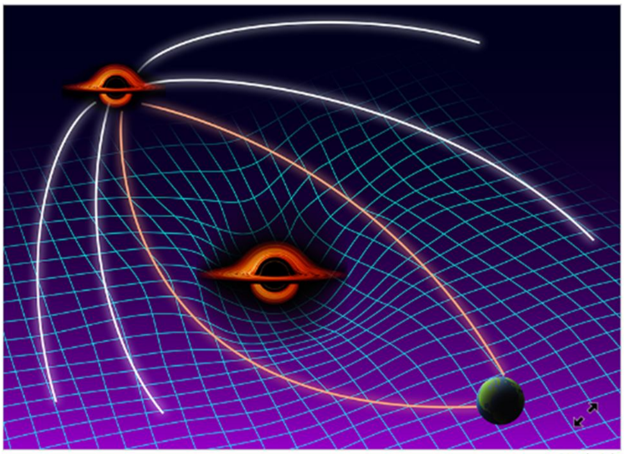
Black holes are one of the most remarkable predictions of Einstein's theory of general relativity, which provides a means to explore them in unstable photon circular orbits. Due to its defining characteristic of the event horizon along with the surrounding photon region, a black hole observes a dark region on the sky known as a shadow [10]. Recently the EHT project, a Global Very Long Baseline Interferometer Array observing at 1,3 mm wavelength, announced the first image of the massive BH at the center of M87, while the corresponding image is of the center of the Milky Way. Collaborated by the Event Horizon Telescope (EHT). The first image revealed the shadow of the supermassive black hole at the center of our own Milky Way Galaxy [1; 3; 4]. The Event Horizon Telescope (EHT) captured the first image of a black hole at the center of the distant galaxy Messier 87 in 2019 [2]. Black hole mimics are possible alternatives to black holes. They are almost observationally similar to black holes but have no horizon. A black hole absorbs all the light that falls on it and cannot be directly imaged, an observer sees a dark spot in the sky where the BH is supposed to be. Due to the strong bending of light rays by the black hole's gravity, both the size and the shape of this spot differ from what we naively expect based on Euclidean geometry from looking at a non-gravitating black ball [8]. To a distant observer, the event horizon casts a relatively large «shadow» with an apparent diameter of about 10 gravitational radii due to the bending of light by the black hole, and this shadow is almost independent of the black hole's spin or orientation. The predicted angular size of the galactic center black hole is about 30 μs, which is only a factor of two smaller than the highest resolution currently achieved by VLBI techniques [6; 7]. The dense central region of our galaxy and many other galaxies generally contains supermassive black holes. The images and shadows created by gravitational lensing of light provide an observational tool to probe the gravitational fields around such compact bodies and discern their nature. The gravitational field near the black hole's event horizon becomes so strong that its outer geometry can have unstable circular photon orbits or optical rings (or photon spheres in the case of statically symmetric black holes), causing photons to bounce indefinitely. jump Increasing the bending value (strong gravitational lensing) A slight perturbation in photons in such unstable orbits can cause them to be absorbed by the black hole or sent to a distant observer. Therefore, the event horizon of a black hole, together with unstable light rings, is expected to produce a characteristic shadow-like image (darker region on a brighter background) of photons emitted from nearby light sources or radiation emitted from accretion. flow around it. Black hole mimics are possible alternatives to black holes. They are almost observationally similar to black holes but have no horizon. A black hole absorbs all the light that falls on it and cannot be directly imaged, an observer sees a dark spot in the sky where the BH is supposed to be. Due to the strong bending of light rays by the black hole's gravity, both the size and the shape of this spot differ from what we naively expect based on Euclidean geometry from looking at a non-gravitating black ball [11]. To a distant observer, the event horizon casts a relatively large «shadow» with an apparent diameter of about 10 gravitational radii due to the bending of light by the black hole, and this shadow is almost independent of the black hole's spin or orientation. The predicted angular size of the galactic center black hole is about 30 μs, which is only a factor of two smaller than the highest resolution currently achieved by VLBI techniques [6]. The dense central region of our galaxy and many other galaxies is generally believed to contain supermassive black holes. The images and shadows created by gravitational lensing of light provide an observational tool for probing the gravitational fields around such compact bodies and discerning their nature. The gravitational field near the black hole's event horizon becomes so strong that its outer geometry can have unstable circular photon orbits or unstable optical rings (or photon spheres in the case of a statically symmetric black hole), causing photons to bounce indefinitely. To grow the bending value (strong gravitational lensing) of a slight perturbation in photons in such unstable orbits can cause them to be absorbed by the black hole or sent to a distant observer. Therefore, the event horizon of a black hole, together with unstable light rings, is expected to produce a characteristic shadow-like image (darker region on a brighter background) of photons emitted from nearby light sources or radiation emitted from accretion. Flow around it. The Event Horizon Telescope (EHT) recently observed this shadow in an image of M87*. However, the observational result of the image of the compact arcuate mass A∗ (Sgr A∗) at the center of our galaxy is still to come [2; 13].

Black hole shadow fundamentals: definition and related concepts

A black hole absorbs all the light that hits it and emits nothing. Therefore, it shows even a naive consideration that an observer will see a dark spot in the sky where the black hole is supposed to be. However, because Strong bending of light rays by BH gravity, both the size and shape of this point with what we naively expect, based on Euclidean geometry, to look at a non-gravitating black ball. About a Spherically symmetric black hole, difference between shadow and imaginary black Euclidean image the hole is only angular in size: the shadow is about two and a half times larger. To rotate Black hole, the shape of the shadow changes: on the one hand, it is deformed and flattened. Size and shape the shadow depends not only on the parameters of the black hole itself, but also on the position of the observer [12].

Measuring a Black Hole Shadow

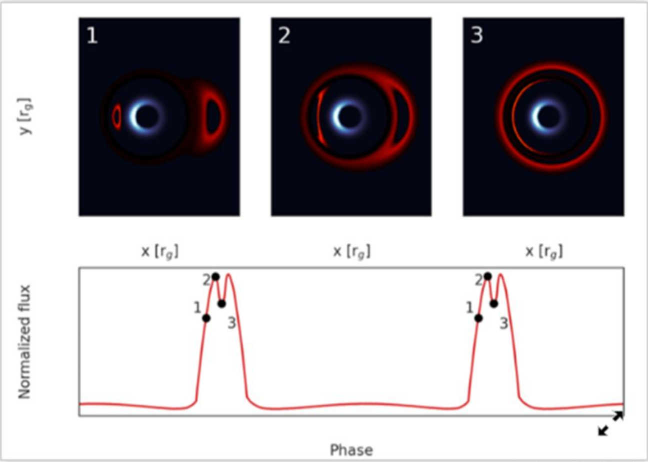
A new method for measuring the shadows cast by a binary black hole could enable astronomers to glean details about these massive systems, as shown in figure 1.



APS/Carin Cain

Figure 1. A cartoon showing the lens of light itself, the supermassive black hole, by a binary system. Jordi Davlar and Zoltan Hyman of Columbia University predict that the work could be used to study binary black holes that are too far from Earth to be probed by other techniques [14]

Unfortunately, for an Earth-bound observer, recovering an image of a black hole shadow is virtually impossible for all but the closest supermassive black holes. This is because most massive black holes are too small to be resolved in the night sky. This is especially problematic for scientists who want to understand how black holes grow, because the first black holes are the ones farthest from Earth and therefore the smallest in the sky. Davlar and Hyman predict that their method could capture the shadows of these intractable objects. In their study, Davlar and Hyman numerically simulated black hole binaries in a variety of different configurations. They used the classical Novikov–Thorne model for gas accretion in discs and chose parameters that were tuned such that their model reproduced the behavior observed in hydrodynamic simulations of shock-heated minidiscs. The duo then used an adaptive ray-tracing code to track the light traveling between the minidiscs and the Earth observer. This numerical implementation allows Davlar and Hyman to easily change the parameters that determine the size and structure of minidiscs, as well as the physical properties of black holes and their orbits.



J. Davelaar and Z. Haiman [2]

Figure 2. Shows projected snapshots (top) and light curves (bottom) of a flare (red) from a binary black hole

Analyzing data from their simulations, Davlar and Hyman looked for binary systems with significant dips in their lensing flares, which are created when one black hole in a binary passes directly behind another (Figure 2). As one of the binary black holes passes behind the other, light from the mini-disc surrounding the more distant black hole is «lensed» by the gravitational field of the closer black hole. This lens produces flare. However, when the lens is placed directly on the farthest black hole and the shadow of that black hole is lensed, the intensity of this flare is reduced. Using estimates for background variability and instrumental noise, Davlar and Hyman estimate that this gradient should be detectable in about 1% of the 150 supermassive black hole binaries found so far [14].

General formula for the radius of shadow of spherically symmetric black holes

We will briefly present the derivation of the shadow formula of an arbitrary spherically symmetric black hole. The metric of a symmetric spherical space-time can be written as follows

$$ds^2 = -A(r)dt^2 + D(r)(d\partial^2 + sine^2\partial d\varphi^2). \tag{1}$$

Any plane in the spherical case can be considered as an equatorial plane, so we can choose $\vartheta = \frac{\pi}{2}$, and thus $p_\vartheta = 0$. The Hamiltonian for light rays has the form

$$H = \frac{1}{2} g^{1k} p_i p_k = \frac{1}{2}\left(-\frac{p_t^2}{A(r)} + \frac{p_r^2}{B(r)} + \frac{p_\varphi^2}{D(r)}\right). \tag{2}$$

The light rays are the solutions to the equations of motion:

$$\dot{P}_i = -\frac{\partial H}{\partial \chi^i}, \quad \dot{\chi}^i = -\frac{\partial H}{\partial p_i}. \tag{3}$$

So that

$$i = -\frac{p_t}{A(r)}\,. \tag{4}$$

$$\varphi = \frac{p_\varphi}{B(r)}. \tag{5}$$

$$\dot{r} = \frac{p_r}{D(r)}\,. \tag{6}$$

From $H = 0$ it follows that

$$0 = -\frac{p_t^2}{A(r)} + \frac{p_r^2}{B(r)} + \frac{p_\varphi^2}{D(r)}. \tag{7}$$

Here a dot designates derivatives with respect to an affine parameter and a prime is for derivatives with respect to r. The momenta p_t and p_φ are constants of motion; $\omega_0 = -pt$.

$$\frac{dr}{d\varphi} = \frac{\dot{r}}{\dot{\varphi}} = \frac{D(r)p_r}{D(r)\,p_\varphi}. \tag{8}$$

Using p_r from $0 = -\frac{p_t^2}{A(r)} + \frac{p_r^2}{B(r)} + \frac{p_\varphi^2}{D(r)}$), we have

$$\frac{dr}{d\varphi} = \pm\frac{\sqrt{D(r)}}{\sqrt{B(r)}}\sqrt{\frac{\omega_0^2}{p_\varphi^2}h(r)^2 - 1}. \tag{9}$$

The function h(r) is defined as follows:

$$h(r)^2 = \frac{D(r)}{A(r)}. \tag{10}$$

A circular light orbit corresponds to zero radial velocity and acceleration, so that $r^{\cdot} = 0$ and $\ddot{r} = 0$ from $i = \frac{p_t}{A(r)}$ it follows that $p_r = 0$, while from $0 = -\frac{p_t^2}{A(r)} + \frac{p_r^2}{B(r)} + \frac{p_\varphi^2}{D(r)}$ we find

$$\frac{\omega_0^2}{A(r)} + \frac{p_\varphi^2}{D(r)} = 0. \tag{11}$$

Differentiating $\dot{r} = \frac{p_r}{D(r)}$ with respect to the affine parameter gives

$$\dot{p}_r = \frac{d}{d\lambda}\left(B(r)\dot{r}\right) = \ddot{r}B(r) + \dot{r}^2 B'(r). \tag{12}$$

Then, the requirement of zero radial velocity and acceleration leads to $\dot{p}_r = 0$, and we find that

$$0 = -\frac{\omega_0^2}{A(r)^2} + \frac{p_\varphi^2 D'(r)}{D(r)^2}. \tag{13}$$

From ($0 = \frac{\omega_0^2}{A(r)} + \frac{p_\varphi^2}{D(r)}$) and($0 = -\frac{\omega_0^2}{A(r)^2} + \frac{p_\varphi^2 D'(r)}{D(r)^2}$) it follows that

$$p_\varphi^2 = D(r)\left(\frac{\omega_0^2}{A(r)}\right) = \frac{D(r)^2}{D'(r)}\left(\frac{\omega_0^2 A'(r)}{D(r)^2}\right). \tag{14}$$

Then, the radius of a photon sphere is a solution to the equation

$$\frac{d}{dr}h(r)^2 = 0. \tag{15}$$

We will use r_O for the position of the observer and α for the angle respectively the radial direction. Then, we have

$$cot\, cot\, \alpha = \frac{\sqrt{g_{rr}}}{\sqrt{g_{\varphi\varphi}}}\frac{dr}{d\varphi}\Big|_{r=r_0} = \frac{\sqrt{B(r)}}{\sqrt{D(r)}}\frac{dr}{d\varphi}\Big|_{r=r_0}. \tag{16}$$

The equation $\frac{dr}{d\varphi} = \pm\frac{\sqrt{D(r)}}{\sqrt{B(r)}}\sqrt{\frac{\omega_0^2}{p_\varphi^2}h(r)^2 - 1}$ can be rewritten in terms of the minimal radius R as follows

$$\frac{dr}{d\varphi} = \pm\frac{\sqrt{D(r)}}{\sqrt{B(r)}}\sqrt{\frac{h^2(r)}{h^2(R)} - 1}. \tag{17}$$

Then we have

$$cot^2\alpha = \frac{h^2(r_0)}{h^2(R)} - 1 \tag{18}$$

and consequently

$$sin^2\alpha = \frac{h(R)^2}{h(r_0)^2}. \tag{19}$$

The angular radius of the shadow is then determined by

$$sin^2\alpha_{sh} = \frac{h(r_{ph})^2}{h(r_0)^2}. \tag{20}$$

With these formulas in hand, we are ready to find the shadow radius for the black hole metric [9].

Black hole shadow in symmergent gravity

To calculate the shadow of the black hole in symmetric gravity, we first obtain the zero geodesic equations and write the Hamiltonian of the moving photon:

$$2H = g^{ij}p_ip_i = 0. \tag{21}$$

Because of the spherically symmetry, in the equatorial plane with $\theta = \frac{\pi}{2}$, the above equation Eq. is written as

$$\frac{1}{2}\left[-\frac{p_t^2}{B(r)} + B(r)p_t^2 + \frac{p_\emptyset^2}{r^2}\right] = 0. \tag{22}$$

The quantities of constants motion p_t and $p_\emptyset$ which are related to energy $-p_t = E$ and angular momentum of the photon $p_\emptyset = L$ can be found as $\dot{p}_t = -\frac{\partial H}{\partial t} = 0$ and $\dot{p}_\emptyset = -\frac{\partial H}{\partial t} = 0$ [5].

Conclusions

One of the most astonishing predictions of Einstein's theory of general relativity is the existence of black holes, which allows for the exploration of them via unstable photon circular orbits. 2019 saw the debut of a black hole imaged by the Event Horizon Telescope (EHT) at the heart of the far-off galaxy Messier 87. A black hole observes a dark area on the sky known as a shadow due to the distinctive feature of the event horizon and the surrounding photon region. Recently, the huge BH at the center of M87 was captured for the first time by the EHT project, a very long global base interferometer array observing at 1,3 mm wavelength. The Milky Way galaxy's nucleus is seen in the corresponding image. An observer perceives a dark area in the sky where the black hole is supposed to be because a black hole absorbs all the light that it encounters and cannot be directly photographed. The size and shape of the spot are different from what we would anticipate from gazing at a non-gravitating black ball based on Euclidean geometry because of the significant bending of light beams caused by the black hole's gravity. Due to the black hole's ability to bend light, the event horizon appears to a distant observer as a relatively large "shadow" with an apparent diameter of about 10 gravitational radii. This shadow is essentially unaffected by the black hole's spin or orientation. This shadow was recently noticed by the Event Horizon Telescope (EHT) in a picture of M87. The observational outcome of the image of the compact arcuate mass A∗ (Sgr A∗) at the galactic center has yet to be released.

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ТЕНЬ ЧЕРНОЙ ДЫРЫ И ЕЕ ИЗМЕРЕНИЕ

В данном исследовании мы рассмотрели такие показатели, как измерение теней черных дыр, общую формулу для расчета радиуса тени симметричных сферических черных дыр и влияние симметричной гравитации на тень черной дыры. Сначала мы получаем нулевые геодезические уравнения, чтобы вычислить тень черной дыры в условиях симметричной гравитации. Имея на руках эти формулы, мы можем определить радиус тени для метрики черной дыры. Мы также кратко обсудим вывод формулы тени для любой заданной сферически-симметричной черной дыры. Теоретически очевидно, что кроме черных дыр и другие компактные объекты могут отбрасывать тени, такие как пустые сингулярности. Мы демонстрируем, как новый метод обнаружения теней, отбрасываемых двойной черной дырой, может позволить астрономам больше узнать об этих гигантских системах. Мы приводим аналитическое вычисление теней для сингулярностей в небытии пространства – времени.

Ключевые слова: *Черная дыра, голая сингулярность, тень.*